MTH241 Fall 2024: Quiz 08

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UID:	

Closed book, no calculator, show your work clearly.

1. (5pt) Evaluate the integral

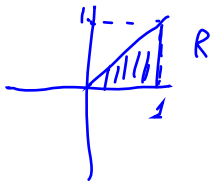
$$\int_0^1 \int_0^x \int_0^{x+y} y dz dy dx.$$

Identify the corresponding solid region D by writing $F_1(x,y)$, $F_2(x,y)$ and R. (Grading: 2pt working; 1pt for evaluation; 2pt for solid region)

$F_1(x,y) = 0$

$F_2(x,y) = x+y$

R is determined by $\int_{x=0}^{x=1} \int_{y=0}^{y=x}$ so

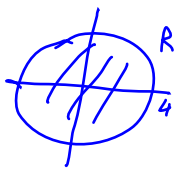


$$\int_0^1 \int_0^x \int_0^{x+y} y \, dz \, dy \, dx = \int_0^1 \int_0^x y(x+y) \, dy \, dx$$

$$= \int_0^1 \int_0^x yx + y^2 \, dy \, dx = \int_0^1 \frac{x^3}{2} + \frac{x^3}{3} \, dx = \int_0^1 \frac{5x^3}{6} = \frac{5}{24}$$

2. (5pt) Let D be the solid region bounded above by the plane $y+z=4$, below by the xy-plane, and on the sides by the cylinder $x^2+y^2=16$. Evaluate $\iiint_D \sqrt{x^2+y^2} dV$. (Grading: 2pt working; 3pt for the integral)

$$\int_R \int_{z=0}^{z=4-y} \sqrt{x^2+y^2} \, dV$$



$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=4} \int_{z=0}^{z=4-r\sin(\theta)} r^2 \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^4 r^2(4 - r\sin(\theta)) \, dr \, d\theta = \int_0^{2\pi} \int_0^4 4r^2 - r^3\sin(\theta) \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^4 4r^2 \, dr \, d\theta = \int_0^{2\pi} \frac{4^4}{3} d\theta = \frac{4^4}{3} \cdot 2\pi$$

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